EBiŚ 2023, 2(80)
DOI: 10.24131/3247.230207
ISNN 1643-8779, e-ISSN 2720-5789

The new epistemology of mathematics and formal sciences in the age of AI. Critical concept kinds and diversity of mental representations

Karolina Tytko*[1], Franci Mangraviti**[2], Natalia Ryłko***[3], Karlygash Nurtazina****[4]
[1]The Pontifical University of John Paul II in Kraków
[2]University of Padova
[3]Cracow University of Technology
[4]L. N. Gumilyov Eurasian National University

In this paper, we aim to show that studies on the new epistemology of mathematics and other sciences are crucial, not only from a theoretical perspective but also in terms of practical aspects, which are particularly significant in the context of teaching and the development of new competences among teachers.

The first aspect addressed here is the age of AI and its potential impact, not just on scientists but on society as a whole. The second aspect focuses on mental imagery, which can be influenced by various factors and, in turn, shape scientific thinking. The third aspect, briefly mentioned, is the issue of the social responsibility of science. Our research approach is grounded in actor-network theory as well as the extended mind thesis.

* E-mail: karolinatytko@interia.com
ORCID: https://orcid.org/0000-0003-2477-9090
** E-mail: legallyfranci@gmail.com
ORCID: https://orcid.org/0000-0003-1537-0883
*** E-mail: natalia.rylko@pk.edu.pl
ORCID: https://orcid.org/0000-0003-0077-5992
**** E-mail: nurtazina_kb@enu.kz
ORCID: https://orcid.org/0000-0002-8086-5631

The main problem analysed here is the relationship between the diversity of mental representations and critical concept kinds in the context of solving mathematical problems (e.g., constructing new structures). Our key conclusion is that this diversity in mental representations can be linked to the presence of critical concept kinds, thereby aiding in the effective construction of new concepts and problem-solving. The need for this diversity in representations can – and should – be cultivated and introduced at the educational level.

Keywords: AI, critical concept kinds, diversity of mental representations, mathematical practice, education

Nowa epistemologia matematyki i nauk formalnych w dobie sztucznej inteligencji. Krytyczne rodzaje pojęć i różnorodność reprezentacji umysłowych

W artykule starano się wykazać, że badania nad nową epistemologią matematyki i innych nauk są kluczowe nie tylko z perspektywy teoretycznej, ale także ze względów praktycznych, które są szczególnie istotne w kontekście dydaktyki i rozwoju nowych kompetencji nauczycieli.

Pierwszy poruszony aspekt dotyczy ery sztucznej inteligencji i jej potencjalnego wpływu nie tylko na naukowców, ale i na całe społeczeństwo. Drugi aspekt skupia się na obrazach mentalnych, na które mogą wpływać różne czynniki, co z kolei może kształtować myślenie naukowe. Trzecim pokrótce omówionym aspektem jest kwestia społecznej odpowiedzialności nauki. Nasze podejście badawcze opiera się na Teorii Aktora-Sieci oraz Teorii Rozszerzonego Umysłu.

Głównym analizowanym problemem jest związek między różnorodnością reprezentacji mentalnych a rodzajami pojęć krytycznych w kontekście rozwiązywania problemów matematycznych (np. konstruowania nowych struktur). Kluczowym wnioskiem jest to, że ta różnorodność

w reprezentacjach mentalnych może być powiązana z obecnością krytycznych rodzajów pojęć, pomagając w ten sposób w skutecznym konstruowaniu nowych koncepcji i rozwiązywaniu problemów. Potrzeba tej różnorodności w reprezentacjach może i powinna być kultywowana i wprowadzana na poziomie edukacyjnym.

Słowa kluczowe: sztuczna inteligencja, rodzaje pojęć krytycznych, różnorodność reprezentacji myślowych, praktyka matematyczna, edukacja

1. Introduction

Mathematics is often regarded as a field of knowledge that is unique, certain, and independent of human influence. Recently, however, there has been a growing emphasis on the need to explore a new epistemology of mathematics. Within the Philosophy of Mathematical Practice (Carter, 2019; Giardino, 2017), issues such as errors in proofs (De Toffoli, 2021), the non-uniqueness of mainstream mathematics (Bloor, 1991; Van Bendegem, 2016; Mangraviti, in press), and the ethical implications and social dimensions of mathematics (Wagner, 2023) have become areas of interest.

To illustrate why the era of AI presents a compelling argument for examining the epistemology of mathematics and other sciences, we will discuss the observed impact of AI tools on society (Olteanu et al., 2019, 2023; Porcaro et al., 2023). In this context, it can be posited that AI tools may influence mental imagery (Nanay, 2021, 2023), which, in turn, can affect cognitive processes in individuals. This aspect is particularly significant from an educational standpoint.

Our research is grounded in actor-network theory (Latour, 2017) and the extended mind thesis (Logan, 2013). The former considers aspects of the human agent, such as naturality, sociality, and the semiotic construction of meaning (Latour, 2017, p. 173). The latter, articulated by Logan, builds on the ideas of McLuhan and Clark, proposing that while we create technological tools, those tools ultimately shape us (Logan, 2013, pp. 267–268). We focus on mental imagery to explore the potential relationship between humans and AI technology. If AI tools influence our mental imagery, and that imagery, in turn, affects our cognitive processes, then we must consider whether certain mathematical patterns embedded in AI can shape individual mental schemas and processes.

The structure of this paper is as follows: the first section explores key aspects of the age of AI in the context of epistemology and the social responsibility of science. The second section discusses the philosophical issue of diversity and introduces the idea of critical concept kinds, drawing inspiration from Dembroff (2019). The final section presents a mathematical example demonstrating the benefits of possessing diverse, critical mathematical concepts.

2. The age of AI

The need for a new epistemology of the sciences is particularly evident in the age of AI. AI tools embed knowledge that can increasingly influence society in powerful and explicit ways. However, this embedded knowledge carries traces of social practices, along with hidden biases and inequalities, which can be further exacerbated as AI continues to shape society.

"Old", traditional epistemology, especially in relation to the formal sciences, is concerned with knowledge that is detached from experience and unrelated to the subjective psychological processes of individuals or the social processes of scientific groups. What holds human knowledge together are justifications and evidence (Bradie & Harms, 2023). The "new" epistemology takes into account the influence of scientific practices (even in the formal sciences) on the construction of theories. In this context, psychology can be seen as a useful, full-fledged analytical tool (Bradie & Harms, 2023). This makes it possible to examine the problem of the relationship between the diversity of mental representations (DMR) and critical concept kinds (CCK).

The "new" epistemology of mathematics differs from the "old" primarily in its approach to complexity and the structure of mathematical knowledge. Traditional epistemology is focused on static structures, viewing mathematical knowledge as fixed and absolute. It assumes mathematical concepts to be uniform and unchanging. In contrast, the new epistemology considers the dynamic, diverse nature of representations and their interactions. It emphasizes that mathematical knowledge is not homogeneous but can take various forms depending on cognitive context and processes. Concepts in this view are shaped by critical, interactive relationships with mental representations, allowing for a more nuanced analysis.

As noted by Porcaro et al. (2023), diversity is not synonymous with fairness. While diversity can lead to equal access to search results that promote fairness, it does not automatically ensure it. In the following discussion, we will explore the potential conditions under which diversity can foster conceptual fairness and non-discrimination and argue why developing such habits of mind is crucial from an educational perspective. In other words, we will propose how cultivating these habits can positively impact social practices.

Even at the level of social data itself, biases are often not directly observable but must be identified through studies in fields like social informatics (Olteanu et al., 2019). This indicates that implicit biases arising from accepted social practices, and their influence on the validity of social data research, are challenging to detect.

A case in point is the use of AI tools in healthcare, where implicit bias can significantly impact social fairness. For example, an AI tool for eye examinations was found to lack diversity in its ophthalmic datasets (Jacoba, 2023). This oversight may have led to the marginalization and exclusion of individuals whose characteristics, such as race, were not adequately represented in the dataset. In this context, ensuring diversity in dataset representation could help prevent bias in AI tools.

3. Diversity and the critical concept kinds

In this section we will ask whether the diversity of mathematical representations implemented in AI can influence and shape individual mental schemas and processes. We propose to consider its impact on having critical concept kinds, along the lines of the critical gender kinds proposed by Dembroff (2019). We also propose to consider the mental schemas as helping in the various domains.

We will focus on the problem of diversity and its relation to the possibility of having a critical concept at the mental level. Work relating to diversity in mathematics and AI can be observed in various areas of philosophical reflection – examples of such work are Mangraviti's proposal of a framework for the philosophy of alternative mathematics or the work of Castillo and others on AI social responsibility (Olteanu et al., 2023; Porcaro et al., 2023). In some of these proposals, diversity serves some positive function.

The notion of diversity discussed here relates to perspective. As emphasized in the social philosophy of science (Longino, 1990), incorporating a broader range of viewpoints increases the likelihood of identifying and eliminating biases in theory construction and selection, thereby leading to a more robust epistemic practice. While this concept is widely accepted in the "soft" sciences, abstract fields like mathematics have traditionally been resistant to such insights. However, this resistance is beginning to change, as the epistemic importance of the social dimensions of mathematics is increasingly acknowledged (Tanswell & Rittberg, 2020; Hunsicker & Rittberg, 2022; Burton, 1995).

The definition of critical concept kinds, based on Dembroff (2019) and Mangraviti (in press), can be presented as follows: a *critical concept kind* is a way of working with a concept which destabilizes mainstream ways of working with the "same" concept. *Critical individual mental practices* are those practices that generate critical concept kinds.

Here we would like to describe the reason why it can be assumed that certain mental habits and patterns (such as the need for a diversity of mental representations and the related need in some cases to have a critical concept kind) can influence individual and social practices. This reason is mental imagery as described by Nanay (2021, 2023).

The definition of mental imagery is as follows: "We use the term 'mental imagery' to refer to representations [. . .] of sensory information without a direct external stimulus" (Pearson et al., 2015, p. 590; Nanay, 2023, pp. 3–4).

The most important aspect of mental imagery is that it is shaped by various external factors that influence past memories and future expectations. Conversely, it significantly impacts overall mental life, emotions, desires, action execution, and multimodal perception (Nanay, 2023).

The second, equally important aspect is that, as Nanay argues, mental imagery can also operate at an unconscious level. In his article, Nanay supports this perspective and discusses specific empirical experiences to illustrate it (Nanay, 2021). Thus, the mental imagery described by Nanay can be both involuntary and unconscious (Nanay, 2023, p. 32).

For the purpose of this article, the most crucial implication of Nanay's work is that the connections between diversity and critical concept kinds, as well as between mental schemas and practices, do indeed exist.

4. The mathematical case study

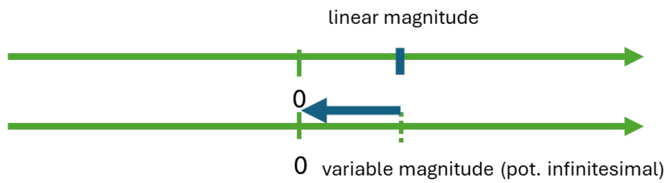
In this section, we will present an example illustrating how the diversity of visual mental representations, along with the possession of a critical concept kind, can enhance problem-solving effectiveness. The example will focus on mathematical cognition, specifically comparing two historical constructions: Cantor's proof against infinitesimals and Dedekind's construction of the set of real numbers. Our analysis will be limited to the perspective of cognitive psychology, with a particular emphasis on visual thinking and mental imagery, as discussed in the works of Bence Nanay (2021, 2023) and Marcus Giaquinto (2007, 2008a, 2008b).

Cantor expressed a bias against actual infinitesimals, distinguishing them from potential infinitesimals (Cantor, 1887, p. 410; Błaszczyk & Fila, 2020a, p. 150). As he explained, from the point of view of pure arithmetic analysis, one should not equate infinitesimals with magnitudes, but only with the way these magnitudes change, i.e., with "variable magnitudes becoming infinitely small" (Cantor, 1882, pp. 156–157; Pogonowski, 2012). In contrast, he described linear numbers in opposition to variable magnitudes as: "numbers which may be regarded as bounded, continuous lengths of straight lines" (Cantor, 1887, p. 407; Ehrlich, 2006, p. 41).

Cantor's attitude described above relates to his platonic assumptions – that the objects of mathematics exist independently of people. Since he always took existing concepts as the starting point of his considerations, he did not attempt to construct them (Cantor, 1883, p. 207).

Figure 1.

The example of the visual representation for the potential infinitesimal.



Source: own elaboration.

This also relates to the methods of the Berlin school (Ferreirós, 1999, pp. 34–36). Cantor's considerations relating to infinitesimals in the context of the real numbers constructed are correct. However, in his proof against infinitesimals he already made a mathematical error, more extensively described elsewhere (Błaszczyk & Fila, 2020a; Cantor, 1887).

Cantor tried to consider the concept of the infinitesimals defined in the context of real numbers, also in the context of the ordinal infinite numbers he introduced. However, he did not formally define the relationship between these elements in a structural sense, but only arbitrarily defined its consequences. We believe that it was determined by him in a visual representation describing infinitesimal quantities as potential, i.e., as variable quantities becoming infinitesimal[1].

Perhaps unconscious mental imagery shaped his approach to this topic. By the fact that the representation may have been the only one accepted (consciously or not), Cantor considered the meaning of the concept determined by it as the only possible one. We argue, therefore, that Cantor lacked critical concept kinds of infinitesimals. This is because he did not attempt to define (introduce) new concepts in isolation from deriving them directly, deductively, from concepts that already existed. Cantor, because of his Platonic beliefs and the methods of the Berlin School, al-

[1] This topic will be further developed in a separate article.

ways took existing concepts and their context as the immutable starting point of his considerations. This also resulted (as we observe in this case) in a lack of variety in the visual representations of certain concepts. Ultimately, it can be argued that Cantor's way of using visual representations was ineffective in this case because it led to the need to prove the Archimedean Axiom not only in the context of real numbers. It led to an erroneous over-generalisation of the relationship between infinitesimals and transfinite ordinals, as Błaszczyk and Fila point out.

This relation is described as follows: when n – natural number, ζ – infinitesimal, Ord – transfinite ordinal, then: $(\forall n \in N)\left(\zeta < \frac{1}{n}\right) = (\forall \nu \in Ord)\ (\forall n \in N)\left(\zeta \cdot \nu < \frac{1}{n}\right)$ (Błaszczyk and Fila, 2020a, p. 176; Cantor, 1887, pp. 407–408). According to this, the product, as Cantor writes, will always be infinitesimal. But, as Błaszczyk and Fila note: "Cantor had never defined the product of infinitesimal and ordinal numbers" (Błaszczyk & Fila, 2020a, p. 176). And they show that this condition in not fulfilled in the field of surreal numbers, where: "every infinite ordinal number α is an infinite element (…) the element α^{-1} is infinitesimal" (Błaszczyk & Fila, 2020a, p. 176). Then, for certain infinitesimal α^{-1} we can find such transfinite ordinal β, that $\beta > \alpha$, and its product can be greater than $1 < \alpha^{-1} \cdot \beta$ (Błaszczyk & Fila, 2020a, p. 176)

Before moving on to Dedekind, it is important to note what Ferreirós (1999) states, that Cantor's and Dedekind's methods were different, as both were educated and worked in different scientific environments. Unlike Cantor, Dedekind did not write much about his philosophical assumptions about mathematics. Nor did he shy away from geometric (visual) intuition, even though he believed that a rigorous, formal introduction of basic structures was important (Dedekind, 1872, pp. 315–16).

The problem that Dedekind solved in constructing the set of real numbers was related to gaps in the set of rational numbers (Dedekind, 1872, p. 325). The original element of his construction of the set of real numbers was the property of continuity, which he defined on the basis of the straight line composed of points (Ferreirós, 1999, p. 133; Dedekind, 1872, p. 322; Błaszczyk & Fila, in press), which is different from the previous use of the term continuum, where it was something not composed of atomic parts (Bell, 2017).

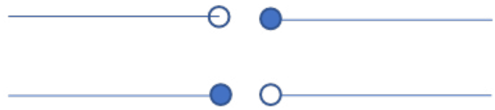
His definition of continuity, we argue, may have been determined by the visual representation of the straight line and cut, which in turn was a response to an earlier mathematical problem (the lack of irrational numbers) and was the purpose of the constructed structure (to define these numbers). We assume that Dedekind used a visual mental representation to define the property of continuity because he referred to a straight line in his work, and he gave a rather intuitive definition of a straight line there at the beginning (Ferreirós, 1999, p. 133; Dedekind, 1872, p. 322).

He wrote:

> If all points in the line are decomposed into two classes, such that each point in the first class is to the left of any point in the second class, then there exists one and only one point, which produces this division of all points in two classes, this cutting of the line in two parts (Ferreirós, 1999, p. 133; Dedekind, 1872, p. 322).

We can say that in Dedekind's case we are dealing with a critical notion of continuity. Dedekind did not limit mental representations only to those associated with the "mainstream" (continuous-discrete opposition), but reached for a new, original representation to illustrate and solve the problem (Błaszczyk & Fila, 2020b, Fig. 2).

Figure 2.
The cuts in the ordered field of real numbers.



Source: Błaszczyk & Fila, 2020b.

His approach in this case allowed him to effectively define the set of real numbers (the continuity property describes it, under some conditions, in a categorical way (Błaszczyk, 2020). Conditions when the property of continuity describes the structure in a categorical way: "The field of real numbers is defined as a commutative ordered field $(\mathbb{F}, +, \cdot, 0\ 1, <)$, in which every Dedekind cut (L, U) of $(\mathbb{F}, <)$ satisfies the following condition: $(\exists x \in \mathbb{F})\ (\forall y \in L)(\forall z \in U)(y \leq x \leq z)$ (C1)" – the continuity property by Dedekind.

The equivalent form of (C1): "The field is Archimedean and every Cauchy (fundamental) sequence $(a_n) \subset \mathbb{F}$ has a limit in $\mathbb{F}$" (Błaszczyk, 2020, p. 148).

Our conclusion is that the diversity of mental representations in Dedekind's case provided him with the critical, effective concept (definition) of continuity. In contrast, Cantor limited his mental (especially visual) representations and created a false relation between the concepts. If Cantor had not considered concepts as existing independently of him, he might have had a more pragmatic approach to them, similar to Dedekind's. That is, he would have considered each concept only in the context of a given structure, he would not have transferred certain properties (like the Archimedean property) from one structure (R) to all structures in mathematics. He might not have limited the visual representations of the concept of infinitesimals to potential ones, because he would not have tried to deny the existence of actual infinitesimals, as described by Kerry, for example (1885, p. 212).

To summarise, we assume that the diversity of mental representations is a necessary but not sufficient condition for forming critical concept kinds. While diversity enables concept formation, specific conceptual structures are also essential, as shown in our Cantor and Dedekind models analysis. The relationship between the diversity of mental representations and the formation of critical concept kinds can be understood as complementary but not inherently complete. DMR plays an enabling role, providing the cognitive flexibility necessary for developing complex concepts by allowing individuals to approach ideas from multiple perspectives. However, diversity alone does not guarantee the formation of CCK; specific conceptual structures and being aware of the purpose of the structure being constructed are also required to guide and organise these representations effectively.

Our analysis of the Cantor and Dedekind models illustrates this interaction. Cantor's proof against actual infinitesimals doesn't provide a critical concept kinds of infinitesimals. This is because of the lack of diversity of mental representations associated with this concept, as he rejected these, if only on the conscious level. In addition, Cantor's proof is not correct because, as we note, there is an inadequate understanding of this concept in the context of the purpose of the structure described in his proof. Whereas Dedekind's focus is on both the mental visual representations and the purpose of the mathematical, introduced structure of real numbers. His approach offers a diversity of mental representations that leads to the formation of a critical concept kinds of continuity. When coupled with structured conceptual frameworks, it provides a categorical description, under certain conditions, of the constructed structure.

5. Conclusion

In summary, we will first address the question: "whether the diversity of mathematical representations implemented in AI can influence and shape individual mental schemes and processes" posed in the section "Diversity and the critical concept kinds". We suggest that the diversity of mathematical representations implemented in AI, which directly relates to what the user can observe, can influence the diversity of mental representations and the creation of critical concept kinds. The use of AI has the potential, first and foremost, to enrich the diversity of users' mental representations. However, we also assume that by enabling the generation of different models, visualisations and mathematical approaches (Mityushev, Nawalaniec & Rylko, 2018) based on representational diversity, AI tools can support the belief that representational diversity is necessary for effective mathematical problem solving.

It is important to further investigate how users perceive and interpret AI-generated outputs and how this affects their individual mental representations. In particular, it is worth investigating whether AI users develop their own diverse representations or simply accept AI-generated outputs as final solutions without critical processing. Analysing these interactions can provide insights into the role of AI in mathematics education and the development of critical thinking skills, especially the formation of critical concept kinds.

The search for a new epistemology in mathematics and other sciences is crucial for identifying and addressing various problems, including those relating to the social responsibility of science and socio-ethical issues. The analyses presented in this article support the view that, particularly in the age of artificial intelligence, the social responsibility of mathematics and its influence on society through its implementation in AI tools must be considered.

Our argument is based on the idea that the diversity of mental representations of concepts can be key to effectively construct new concepts, relationships, and structures. Therefore, the diversity of mental representations in mathematical practice may play a significant role in effective problem-solving and the construction of mathematical structures. This notion is closely linked to the ability to develop critical concept kinds. A primary consideration here is that the mental representations possessed by each individual can be shaped by AI tools and the knowledge embedded within them.

Given the profound impact of artificial intelligence on society, it becomes important to explore whether it is possible to cultivate the need for diversity in the representation of certain concepts and practical problems by designing suitable schemes at the theoretical level. We hope this article can serve both as a starting point and a motivation for further research into the new epistemology of mathematics and other sciences. This pursuit is not solely for theoretical reasons but also for socio-ethical ones, such as striving for a just and non-discriminatory society, especially given the influence of AI tools on mental imagery, even through mathematical problems.

Acknowledgements

Tytko acknowledges financial support from a Centre for Science Studies at Aarhus University, under the EPSA Fellowship Programme.

Mangraviti acknowledges the financial support of a PNRR grant, under the European Union's NextGenerationEU research and innovation programme.

Bibliography

Bell, J. L. (2017). Continuity and Infinitesimals. In E. N. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy*, Summer 2017. https://plato.stanford.edu/archives/sum2017/entries/continuity/

Błaszczyk, P. (2020). What theory of infinity should be taught and how? *Mathematics Teaching Research Journal*, *12*(2). http://www.hostos.cuny.edu/mtrj/

Błaszczyk, P., & Fila, M. (2020a). Cantor on infinitesimals. Historical and modern perspective. *Bulletin of the Section of Logic*, *49*(2), 149–79. http://dx.doi.org/10.18778/0138-0680.2020.09

Błaszczyk, P., & Fila, M. (2020b). Modes of Continuity in Diagram for Intermediate Value Theorem. In A-V. Pietarinen, S. Linker, P. Chapman, J. Corter, V. Giardino, & L. Bosveld-de Smet (Eds.), *Diagrammatic Representation and Inference.* (pp. 34–49). Springer.

Błaszczyk, P., & Fila, M. (in press). On Bolzano and Greek Concepts of Continuity. In B. Sriraman (Ed.), *Handbook of the History and Philosophy of Mathematical Practice*. Springer International Publishing.

Bradie, M., & Harms, W. (2023 Spring). Evolutionary Epistemology. In E. N. Zalta, & U. Nodelman (Eds.), *The Stanford Encyclopedia of Philosophy.* Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/archives/spr2023/entries/epistemology-evolutionary/

Burton, L. (1995). Moving towards a feminist epistemology of mathematics. *Educ Stud Math*, *28*(3), 275–291.

Cantor, G. (1882). Über unendliche, lineare Punktmannichfaltigkeiten 3 (Mathematische Annalen 20 (1), 113–121). In E. Zermelo (Ed.), *Gesammelte Abhandlungen mathematischen und philosophischen Inhalts* (pp. 149–157). Verlag von Julius Springer. https://gdz.sub.uni-goettingen.de/download/pdf/PPN237853094/PPN237853094.pdf

Cantor, G. (1883). Über unendliche lineare Punktmannichfaltigkeiten 5 /Grundlagen einer allgemeinen Mannigfaltigkeitslehre (Mathematische Annalen 21 (4), 545–591). In E. Zermelo (Ed.), *Gesammelte Abhandlungen mathematischen und philosophischen Inhalts* (pp. 165–209). Verlag von Julius Springer. https://gdz.sub.uni-goettingen.de/download/pdf/PPN237853094/PPN237853094.pdf

Cantor, G. (1887). Mitteilungen zur Lehre vom Transfiniten,(Zeitschrift für Philosophie und philosophische Kritik 91, 81–125). In E. Zermelo (Ed.), *Gesammelte Abhandlungen mathematischen und philosophischen Inhalts* (pp. 378–439). Verlag von Julius Springer. https://gdz.sub.uni-goettingen.de/download/pdf/PPN237853094/PPN237853094.pdf

Dedekind, R. (1872). Stetigkeit und irrationale Zahlen. In R. Fricke, E. Noether and O. Ore (Eds.), *Gesammelte mathematische Werke* (pp. 315–334). Verlag von Friedr. Vieweg&Sohn Akt.-Ges. https://gdz.sub.uni-goettingen.de/download/pdf/PPN23569441X/PPN23569441X.pdf

Dembroff, R. (2019). Beyond binary: genderqueer as critical gender kind. *Philosopher's Imprint*, *20*(9), 1–23.

Ehrlich, P. (2006). The Rise of non-Archimedean Mathematics and the Roots of a Misconception I: The Emergence of non-Archimedean Systems of Magnitudes. *Arch. Hist. Exact Sci.*, 60, 1–121.

Ferreirós, J. (1999). *Labyrinth of Thought. A history of set theory and its role in modern mathematics*. Birkhäuser.

Giaquinto, M. (2007). *Visual Thinking in Mathematics: An Epistemological Study*. Oxford University Press.

Giaquinto, M. (2008a). Cognition of Structure. In P. Mancosu (Ed.), *The Philosophy of Mathematical Practice.* (pp. 43–64). Oxford University Press.

Giaquinto, M. (2008b). Visualizing in Mathematics. In P. Mancosu (Ed.), *The Philosophy of Mathematical Practice.* (pp. 22–42). Oxford University Press.

Hunsicker, E. and Rittberg, C. (2022). On the Epistemological Relevance of Social Power and Justice in Mathematics. *Axiomathes*, *32*(3), 1147–1168. https://doi.org/10.1007/s10516-022-09629-z

Mangraviti, F. (in press). Critical math kinds: a framework for the philosophy of alternative mathematics. *Erkenntnis.*

Kerry, B. (1885). Ueber G. Cantors Mannigfaltigkeitsuntersuchungen. *Vierteljahresschrift für wissensch. Philos.*, 9, 191–232.

Mityushev, V., Nawalaniec, W., & Ryłko, N. (2018). *Introduction to Mathematical Modeling and Computer Simulations*. CRC – Taylor & Francis.

Nanay, B. (2021). Unconscious mental imagery. *Phil. Trans. R. Soc. B*, *376*(1817), 20190689. https://doi.org/10.1098/rstb.2019.0689

Nanay, B. (2023). *Mental Imagery: Philosophy, Psychology, Neuroscience* (1st ed.). Oxford University Press. https://doi.org/10.1093/oso/9780198809500.001.0001

Olteanu, A., Ekstrand, M., Castillo, C., & Suh, J. (2023). Responsible AI Research Needs Impact Statements Too. *ArXiv* abs/2311.11776, 1–9. https://api.semanticscholar.org/CorpusID:265294449

Pearson, J., Naselaris, T., Holmes, E. A., & Kosslyn, S. M. (2015). Mental imagery: functional mechanisms and clinical applications. *Trends Cogn. Sci.*, 19, 590–602. https://doi.org/10. 1016/j.tics.2015.08.003

Pogonowski, J. (2012). O nieskończonych liniowych rozmaitościach punktowych (G. Cantor, Ueber unendliche, lineare Punktmannichfaltigkeiten. Mathematische Annalen XX, 113–121. 1882). Eudoksos-Dedekind-Conway. *2012*. http://www.eudoxos.pl/tlumaczenia/

Porcaro, L., Castillo, C., Gómez, E., & Vinagre, J. (2023). Fairness and Diversity in Information Access Systems. *arXiv 2305.09319*.

Tanswell, F., & Rittberg, C. (2020). Epistemic injustice in mathematics education. *ZDM: The International Journal on Mathematics Education*, *52*(6), 1199–1210. https://doi.org/10.1007/s11858-020-01174-6